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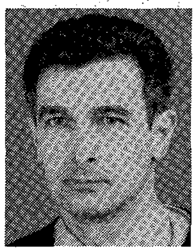

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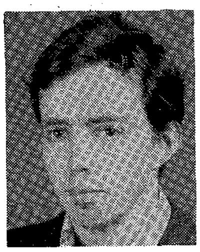

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The Nonlinear Coherent Coupler

STEPHEN M. JENSEN

***Abstract*—This paper discusses the nonlinear coherent coupler (NLCC), a device useful for optical processing, but not bistable.**

This device utilizes the coherent interaction of two optical waveguides placed in close proximity. Because of the evanescent field overlap, these waveguides periodically exchange power. Nonlinear interactions modify the exchange of power and lead to strongly nonlinear transmission characteristics.

INTRODUCTION

RECENTLY, there has been great interest in the possibility of using optical devices for ultra-high-speed data processing. Many devices have been studied; of these the ones receiving the greatest attention are bistable optical devices [1]. Bistability, however, is not necessary in many logic operations. This paper discusses the nonlinear coherent coupler (NLCC), a device useful for optical processing, but not bistable.

This device, first described in 1980 [2], utilizes the coherent interaction of two optical waveguides placed in close proximity. Because of the evanescent field overlap, these waveguides periodically exchange power. Nonlinear interactions modify the exchange of power and lead to strongly nonlinear transmission characteristics which may be utilized in optical processing applications.

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Fig. 1 shows a schematic of the NLCC device. The device is simply two waveguides placed adjacently so that they will couple to one another. The nonlinear material covers the region of interaction between the two waveguides. This nonlinearity may be due to the intrinsic nonlinearity of the substrate material, nonlinearity induced by doping of the substrate material, or by overlayers of highly nonlinear materials.

We have obtained an analytic solution for the NLCC response. To begin the analysis, we assume that the response of the medium, evident as a polarization, may be considered in two parts. The unperturbed linear portion $\bar{P}_o = 4\pi(\epsilon - 1)\bar{E}$ contains contributions from a single isolated waveguide. The perturbing polarization $\bar{P}'$ contains linear contributions from a waveguide placed in close proximity and nonlinear contributions due to the nonlinear response of the material. Using Maxwell's equations, in Gaussian units, one finds that

$$\bar{\nabla}_z \times \bar{E}_t(\bar{r}) + i\frac{c}{\omega}\bar{\nabla}_t \times \left\{\frac{1}{\epsilon}\bar{\nabla}_t \times \bar{H}_t(\bar{r})\right\} - i\frac{\omega}{c}\bar{H}_t(\bar{r}) = 4\pi\bar{\nabla}_t \times \frac{1}{\epsilon}\bar{P}'_z(\bar{r}) \tag{1}$$

and

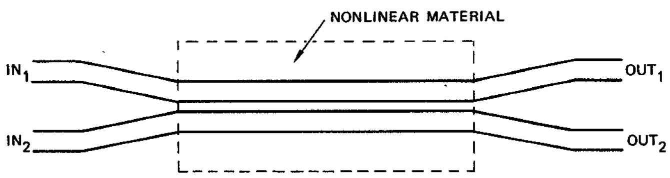

Fig. 1. Schematic of an integrated optic nonlinear coherent coupler.

$$\bar{\nabla}_z \times \bar{H}_t(\bar{r}) - i\frac{c}{\omega}\bar{\nabla}_t \times \{\bar{\nabla}_t \times \bar{E}_t(\bar{r})\} + i\frac{\omega}{c}\bar{E}_t(\bar{r})$$

$$= -i4\pi\frac{\omega}{c}\bar{P}'_t(\bar{r}) \tag{2}$$

where $\bar{\nabla}_t$ and $\bar{\nabla}_z$ represent the transverse and longitudinal components of the curl operator, $\bar{E}_t(\bar{r})$ and $\bar{H}_t(\bar{r})$ are the transverse components of the electric and magnetic fields, c is the speed of light in vacuum, and all the fields are assumed to be harmonic at frequency ω. In (1) and (2) the perturbing polarization $\bar{P}'(\bar{r})$ appears as a source term which modifies the fields $\bar{E}(\bar{r})$ and $\bar{H}(\bar{r})$. Using the standard coupled mode technique of expansion in ideal modes [3]-[5], (1) and (2) may be reduced to the form

$$\Sigma_\nu \{\partial a_\nu(z)/\partial z\}\,\hat{z} \times E_t^\nu(\bar{r}) = 4\pi\bar{\nabla}_t \times \frac{1}{\epsilon}\bar{P}'_z(\bar{r}) \tag{3}$$

$$\Sigma_\nu \{\partial a_\nu(z)/\partial z\}\,\hat{z} \times \bar{H}_t^\nu(\bar{r}) = -i4\pi\frac{\omega}{c}\bar{P}'_t(\bar{r}) \tag{4}$$

where ν represents the νth ideal mode of the waveguide, $a_\nu(z)$ is the amplitude of the νth mode, $\hat{z}$ is a unit vector along the longitudinal (propagation) direction, and the sum extends over all bound and radiation modes of the structure. To proceed, we must utilize the orthonormality relation for the waveguide modes

$$\int dx\,dy\,\hat{z}\cdot\bar{E}_t^\nu(\bar{r}) \times \bar{H}_t^{\nu'}(\bar{r})^* = \frac{2\pi}{c}P_o\delta_{\nu\nu'} \tag{5}$$

where the asterisk indicates a complex conjugate, P_o is the normalization power, and $\delta_{\nu\nu'}$ is the Kronecker delta function. By using the orthonormality relation, (3) and (4) may be reduced to

$$-i\partial a_\nu(z)/\partial z = \frac{\omega}{P_o}\int dx\,dy\,\bar{E}^\nu(\bar{r})^* \cdot P'(\bar{r}). \tag{6}$$

By using (6) one may derive the nonlinear coupled mode equations by properly identifying the contributions to the perturbing polarization $\bar{P}'(\bar{r})$. Linear contributions to the perturbing polarization arise from the overlap of the modal field with an adjacent waveguide and from the presence of any mode of the adjacent guide. Nonlinear contributions arise due to the mode interacting with the material by itself or in conjunction with a mode in the adjacent waveguide.

We wish to analyze the case where two single-mode waveguides are placed in close proximity and configured to run parallel over an interaction length (see Fig. 1). Equation (6) is used to predict the behavior of the mode in each waveguide; one finds that

$$-i\partial a/\partial z = Q_1 a + Q_2 a' + (Q_3|a|^2 + 2Q_4|a'|^2)\,a \tag{7}$$

and

$$-i\partial a'/\partial z = Q_1 a' + Q_2 a + (Q_3|a'|^2 + 2Q_4|a|^2)\,a' \tag{8}$$

where a and a' are the complex normalized amplitudes of the modes and Q_1-Q_4 are the coupling coefficients defined as follows:

$$Q_1 = \frac{\omega}{4\pi P_o}\int dx\,dy\,\delta|E|^2, \tag{9}$$

$$Q_2 = \frac{\omega}{4\pi P_o}\int dx\,dy\,(\epsilon+\delta)EE'^*, \tag{10}$$

$$Q_3 = \frac{n_o n_2\omega}{\pi P_o}\int dx\,dy\,|E|^4, \tag{11}$$

$$Q_4 = \frac{n_o n_2\omega}{\pi P_o}\int dx\,dy\,|E|^2|E'|^2 \tag{12}$$

where $\bar{E}$ and $\bar{E}'$ are the electric field of the two modes, ϵ is the unperturbed susceptibility of one guide, δ is the linear perturbing susceptibility of that guide, and n_2 is the nonlinear refractive index [6]. In (7) and (8), the two coupled waveguides are assumed to be identical. Failing to make this assumption introduces a phase mismatch term into Q_2. The spatial dependence of Q_2 makes the analysis very complicated and the results cannot be expressed as standard elliptic integrals. We also choose the relative phase of E and E' so that the coefficient Q_2 is real [see (10)].

As was discussed above, one can identify each term in (7) and (8) with a particular contribution of the perturbing polarization. The terms involving Q_1 arise from the overlap of the mode field with the adjacent waveguide, they serve only to modify the propagation constant of the mode. The Q_2 terms arise because of the presence of a mode in the adjacent guide and lead to linear coupling between the waveguides. Terms with Q_3 are the strongest nonlinear terms and arise from the nonlinear interaction of a mode with itself. They are equivalent to the self-phase-modulation and self-focusing terms in free space nonlinear optics. The terms involving Q_4 arise from the nonlinear interaction of one mode with the mode in the adjacent guide. In general, there are additional nonlinear terms. These additional terms all have overlap integrals of the form $\int dx\,dy E\cdot E'^*|E|^2$, and are orders of magnitude smaller in the cases of interest.

To analyze (7) and (8), we make the following substitutions:

$$a = Ae^{i(\phi + Q_1 z)} \tag{13}$$

and

$$a' = A'e^{i(\phi' + Q_1 z)} \tag{14}$$

where A, A', ϕ, and ϕ' are real functions of z. Substituting (13) and (14) into (7) and (8) gives four real equations for the four unknowns. From these four equations we find two constants of the motion, the total power

$$P_t = A^2 + A'^2 \tag{15}$$

and

$$\Gamma = 4AA' \cos(\Psi) - \frac{2(Q_3 - 2Q_4)}{Q_2} A^2 A'^2 \tag{16}$$

where

$$\Psi = \phi - \phi'. \tag{17}$$

At this point it is possible to derive an equation for the power propagating in one waveguide

$$(\partial P/\partial z)^2 = Q_2 \{4Q_2 - \Gamma(Q_3 - 2Q_4)\} P(P_t - P) - \tfrac{1}{4}\Gamma^2 Q_2^2 - (Q_3 - 2Q_4)^2 P^2 (P_t - P)^2. \tag{18}$$

Equation (18) integrates as an elliptic integral [7]. The solution is

$$P(Z) = \frac{1}{2} P_c + \frac{\gamma\delta}{\sqrt{\gamma^2 + \delta^2}} P_c \, sd\,\{Z\sqrt{\gamma^2 + \delta^2} + F(\phi_o|m)|m\} \tag{19}$$

where

$$Z = Q_2 z \tag{20}$$

$$(\gamma P_c)^2 = -4P_t^2 + 2P_c(P_c - \Gamma) + 2P_c\sqrt{P_c - 2\Gamma P_c} \tag{21}$$

$$(\delta P_c)^2 = 4P_t^2 - 2P_c(P_c - \Gamma) + 2P_c\sqrt{P_c - 2\Gamma P_c} \tag{22}$$

$$\sin^2(\phi_o) = \frac{(\gamma^2 + \delta^2)\,\{P(0) - \frac{1}{2}P_t\}^2}{\delta^2\{[P(0) - \frac{1}{2}P_t]^2 + P_c^2\gamma^2/16\}} \tag{23}$$

$$m = \delta^2/(\gamma^2 + \delta^2) \tag{24}$$

and P_c is the critical power defined by

$$P_c = 4Q_2/(Q_3 - 2Q_4). \tag{25}$$

$F(\phi_o|m)$ is an elliptic integral of the first kind, and $sd(\theta|m)$ is a Jacobi elliptic function [7]. There is a special case, of interest to optical data processing, for which these equations simplify. This is the case where all of the power is initially launched into one waveguide, i.e., $P(0) = P_t$. In this limit, $\Gamma = 0$ and (19) becomes

$$P_1(Z) = P_1(0)\{1 + cn(2Z|m)\}/2 \tag{26}$$

where

$$m = P(0)^2/P_c^2 \tag{27}$$

and $cn(\phi|m)$ is a Jacobi elliptic function [7]. This elliptic function is periodic with a period of $4K(m)$, where $K(m)$ is a complete elliptic integral of the first kind [7]. In the limit of very small input intensities $m \simeq 0$, (26) becomes

$$P_1(Z) = P_1(0)\{1 + \cos(2Z)\}/2. \tag{28}$$

This is the well-known solution for a linear coherent coupler. The optical power transfers back and forth between the waveguides with a transfer length of $z = \pi/2Q_2$. As the input intensity is increased, the parameter m grows. This leads to increases in $K(m)$, and hence, the period of the elliptic function $cn(\phi|m)$.

Equation (26) is plotted in Fig. 2; the figure shows the amount of power that remains in waveguide 1 plotted as a function of position along the coupler. Fig. 2 shows two distinct types of solutions. For low input powers ($P_1(0) < P_c$), the device acts as a conventional phase-matched coupler; at some point ($Z/\pi \sim 0.5$), the light is switched from guide 1 to guide 2. This is the familiar crossed state of a phase-matched coupler or $\Delta\beta$ reversal switch. The operation of this device is closely related to the $\Delta\beta$ reversal switch. Initially, all of the light is in guide 1, causing a nonlinear detuning of guide 1. As the light slowly couples into guide 2, it starts detuning that guide as well. When the power in each guide is equal, the detuning induced in each waveguide is equivalent. Hence, there is no net detuning between the guides. As more power couples into guide 2, the detuning reverses, just as in a $\Delta\beta$ reversal switch, and drives the device into a fully crossed state. At high input intensities ($P_1(0) \geq P_c$), the device does not go into the crossed state. This is because the detuning induced by the nonlinear index drives the two waveguides out of phase matching, and the 50/50 power distribution point is never achieved; hence, the phase is not reversed and a crossed state is not achieved.

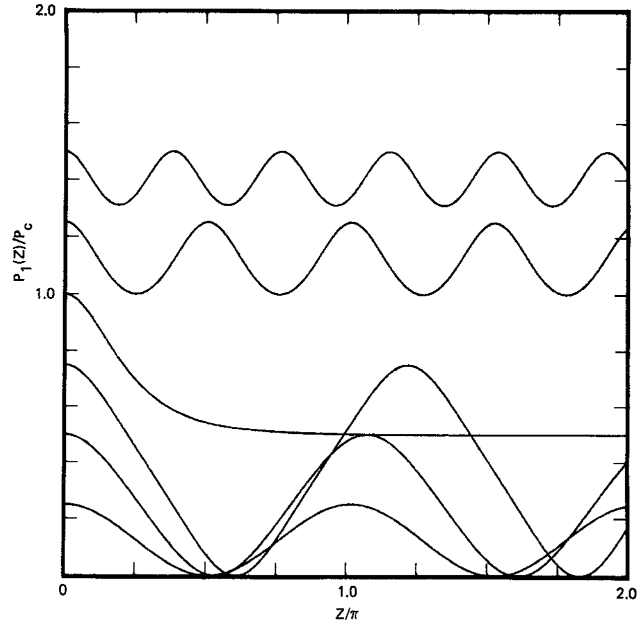

Fig. 2. The amount of power remaining in guide 1 as it propagates along the coupler. The figure shows distinctly different solutions for input powers $P_1(0) < P_c$ and for input powers $>P_c$.

Fig. 3 shows the input/output characteristics of several NLCC's. Each curve shows the normalized output of guide 1 as a function of the normalized input to guide 1 for a device of fixed length. The two curves corresponding to $Z = \pi/2$ and $Z = 3\pi/2$ represent devices that are initially (low inputs) in a crossed state. As the input is increased, some of the light is no longer coupled into guide 2. At sufficiently large inputs, the power remains in guide 1. These operating characteristics would be useful for constructing optical AND gates. On the other hand, the output of guide 2 exhibits the characteristics of an XOR gate. The curves corresponding to $Z = \pi$ and $Z = 2\pi$ exhibit the same type of solution, but with the roles of guides 1 and 2 switched.

It can be shown that

$$cn(2nK(m)|m) = (-1)^n \tag{29}$$

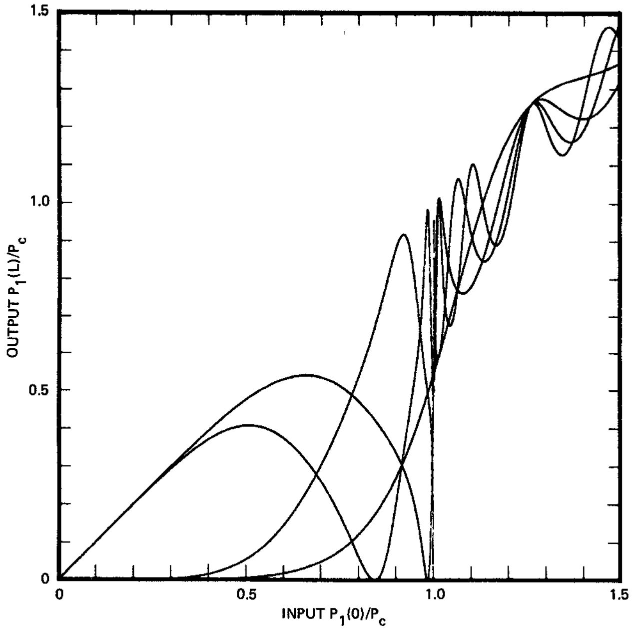

Fig. 3. Output versus input for NLCC's of fixed length.

where

$$n = 0, 1, 2, 3, \cdots.$$

It is therefore possible to predict the powers necessary to operate the NLCC as an optical logic gate. Let us consider a device that is initially (low-input intensity) in a parallel state. That is, we choose the length

$$l = n/Q_2 (n = 1, 2, 3, \cdots). \tag{30}$$

By increasing the input power to a high value we must drive the device from the parallel state into the crossed state. This is accomplished by changing the period from 2π to $2\pi/(1 - 1/2n)$, implying a change of $K(m)$ from its value of $\pi/2$ at $m = 0$ to a value of $\pi/2(1 - 1/2n)$. Values of m and $K(m)$ given for the first few values of n are expressed in Table I. The last column shows the input intensity calculated using (22). To evaluate the actual input intensity we must evaluate the coupling coefficients given by (8)-(11). To this end we assume that the waveguide "mode" may be approximated by having a plane wave within the guide and having zero field elsewhere. Then

$$Q_3 = (8\pi^2 n_2/\lambda n_o c) P_o/A \tag{31}$$

and

$$Q_4 = 0$$

where A is the cross-sectional area of the waveguide. Using (25) the critical power is given by

$$P_c = Q_2 A(\lambda n_o c/2\pi^2 n_2). \tag{32}$$

Assuming that $Q_2 = 3.14 \cdot \text{cm}^{-1}$ (0.5 cm exchange length, $A = 3$ μm^2, $\lambda = 1.06$ μm, $n_o = 3.5$, and $n_2 = 5 \times 10^{-10}$ ESU, then

TABLE I

n	$K(m)$	m	$P_1(0)/P_c$
1	3.142	0.9690	0.985
2	2.094	0.7126	0.844
3	1.885	0.5350	0.732

the critical input power is ~10.6 W. Using values from the table above, we find that for a 1.0 cm device ($n = 1$) the necessary input power is 10.5 W; for a 2.0 cm device ($n = 2$) the input power is 9.0 W; and for a 3.0 cm device ($n = 3$) the input power is 7.8 W.

Although these powers appear quite high, it must be pointed out that this device is capable of exceedingly fast switch times. In fact, since the fields within the device interact in a spatially and temporally local fashion, the switch time of this device is not limited by propagation time. When optical pulse lengths are shorter than the device length, several pulses can, simultaneously and independently propagate through the device in a serial fashion. The device will, in effect, be processing all of the pulses simultaneously. In this limit the switch time of the device will be limited only by the nonlinear response time ($\sim 10^{-14}$ s in many materials). Switch powers can of course be reduced by utilizing materials with larger nonlinearities (generally, also slower). Recent, nonlinearities as large as 0.1 ESU have been observed [8]. By utilizing these materials, switch powers of nanowatts may be achieved.

Conclusion

In summary we have described a simple device capable of performing optical processing functions and very fast data rates ($\sim 10^{12}$ bits/s). This device may be fabricated by utilizing conventional fabrication techniques developed for integrated optic switches and modulators.

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Stephen M. Jensen, photograph and biography not available at the time of publication.